Hints for homogeneous and particular solutions

[Homework 73cjk moved to Monday...]

eg $a_{n+1} = 7a_n - 6a_{n-1} + 6^n + 3^n + 4n + 8$

$a_1 = 3, a_2 = 6$

Homogeneous solution: $r^2 - 7r + 6 = 0$

$(r-1)(r-6) = 0$

gen homog solution $a_n = C_1(1)^n + C_2 6^n$

$a_n = C_1 + C_2 \cdot 6^n$

Initial guess for particular solution: (polynomial part)

try $a_n = k_1 6^n + k_2 3^n + k_3 n + k_4$ — will not work.

($k_1 6^n$: homogen solution; $k_3 n + k_4$: homog. soln)

Better guess that will work

$a_n = k_1 n \cdot 6^n + k_2 3^n + k_3 n^2 + k_4 n$ will work

↳ plug to whole recursive formula → solve for k_1, k_2, k_3, k_4.

Call particular solution answer a_n^p

Whole gen solution. $a_n = C_1 + C_2 \cdot 6^n + a_n^p$

Then plug IC's in.

Type some Sage code below and press Evaluate.

```
from sympy import Function, rsolve
a = Function('a')
var('n')
f = a(n+1)-7*a(n)+6*a(n-1)-6^n-3^n-4*n-8
ans = rsolve(f,a(n))
show(ans)
realans = rsolve(f,a(n),{a(1):3,a(2):6})
show(simplify(realans))
```

Evaluate

```
-3**n/2 + 6**n*C1 + 6**n*n/5 + C0 - 2*n*(5*n + 27)/25
-3**n/2 + 6**n*n/5 - 16*6**n/125 - 2*n*(5*n + 27)/25 + 1657/250
```

General Inclusion Exclusion for 4 sets

$$S = S_1 \cup S_2 \cup S_3 \cup S_4$$

$$|S| = |S_1| + |S_2| + |S_3| + |S_4| - |S_1 \cap S_2| - |S_2 \cap S_3| - |S_3 \cap S_4|$$
$$- |S_1 \cap S_3| - |S_2 \cap S_4| - |S_1 \cap S_4|$$
$$+ |S_1 \cap S_2 \cap S_3| + |S_1 \cap S_3 \cap S_4| + |S_2 \cap S_3 \cap S_4| + |S_1 \cap S_2 \cap S_4|$$
$$- |S_1 \cap S_2 \cap S_3 \cap S_4|.$$

Example We know that TCU has 700 students who speak Hindi, 500 students who speak Ukrainian, 5000 students who speak Spanish. There are 300 students who speak both Hindi & Ukrainian. There are 100 students who speak Hindi & Spanish, and there are 200 students who speak Spanish and Ukrainian. There are 59 students who speak Spanish, Ukranian, and Hindi.

Question: How many students speak Spanish, Hindi, or Ukranian?

Solution: $|S \text{ or } H \text{ or } U| = |S| + |H| + |U|$

$- |S \cap H| - |H \cap U| - |S \cap U|$

$+ |S \cap H \cap U|$

$= 5000 + 700 + 500$

$- 100 - 300 - 200$

$+ 59$

$= 6200 - 600 + 59$

$= \boxed{5659 \text{ students}}$